NUCLEATE BOILING OF A LIQUID

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A balance method is described for the analysis of nucleate pool boiling of a liquid.

The problem of analyzing the local characteristics of the nucleate regime has been stated in several papers on boiling [1-12, 14]. An important consideration here is the fact that the equations of motion of the system must be integrated simultaneously with the equation for the bubble-size distribution function f. If the disintegration and coalescence of bubbles are disregarded, the one-dimensional equation for f can be written in the form

$$\varphi_t + (\varphi u_1)_x + (\varphi \dot{v})_v = n_c P_c \varphi_c; \quad \varphi(t,\,0,\,v) = \frac{n_0 A_0}{u_1}\varphi_0; \tag{1}$$

$\varphi = fA$ is the number of particles of a given diameter per unit length; A and P_c are the cross section and perimeter; n_c, n_0, φ_c, and φ_0 are the fluxes and distributions of the bubbles at the confining walls and bottom of the volume. Integrating Eq. (1) from 0 to x_1, we obtain the following equation for the bubble distribution averaged over the volume:

$$\begin{gathered}\psi_t + (\dot{v}\psi)_v + \frac{\psi}{t_1} = Sn_c\bar{\varphi}_c; \\ t_1 = \frac{x_1}{u_1}; \quad Sn_c\bar{\varphi}_c = \int\limits_0^{x_1} n_c P_c \varphi_c dx + n_0 A_0 \varphi_0; \quad \psi = \int\limits_0^{x_1} \varphi dx.\end{gathered} \tag{2}$$

The growth rate of the moving bubbles can be determined from the expressions [7-9]

$$\dot{v} = \alpha v^{\frac{1}{2}}; \quad \alpha = \sqrt{24}\,(au_{s1})^{\frac{1}{2}}\mathrm{Ja}; \quad \mathrm{Ja} = \frac{c_p \rho \Delta T}{\rho_1 h_{\mathrm{vap}}}. \tag{3}$$

The solution of (2) with regard for (3) with ΔT = const, the initial condition $\psi(0,\,v) = \psi_0(v)$ and $\bar{\varphi}_c = \delta(v - v_0)$ is

$$\begin{aligned}\psi(t,\,v) = \xi^{-\frac{1}{2}}\Bigg(&\psi_0\left[v_0\left[\xi^{\frac{1}{2}} - \frac{t}{A_1}\right]^2\right]\exp\left(-\frac{t}{t_1}\right)\left(\xi^{\frac{1}{2}} - \frac{t}{A_1}\right) \\ &- \frac{Sn_c}{\alpha v_0^{\frac{1}{2}}}\,\theta\left(A_1\left(\xi^{\frac{1}{2}} - 1\right) - t\right)\exp\left(D\left(1 - \xi^{\frac{1}{2}}\right)\right) \\ &+ \frac{Sn_c}{\alpha v_0^{\frac{1}{2}}}\,\theta(\xi - 1)\exp\left(D\left(1 - \xi^{\frac{1}{2}}\right)\right)\Bigg),\end{aligned} \tag{4}$$

where $A_1 = 2v_0^{1/2}/\alpha$; $D = A_1/t_1$; $\xi = v/v_0$. The total number of bubbles in the volume and the average vapor content can be determined from the equations

$$n(t) = \int\limits_0^\infty \psi dv; \quad V_1 = c(t)\,V_\Sigma = \int \psi v dv. \tag{5}$$

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It is clear from (4) and (5) that for $\psi_0 = 0$, $n \sim t$ and $c \sim t^3$ when $t \gg 1$. We also consider another technique for the solution of the vapor-content problem. Integrating (1) with respect to vdv, we obtain the following continuity equation for the vapor phase:

$$\frac{\partial}{\partial t}(cA) + c_1 \frac{\partial}{\partial x}(\bar{u}_1 cA) = \int \dot{v}\varphi dv + \int n_c P_c \varphi_c v dv \equiv H_1 + H_2 \equiv H. \tag{6}$$

Here $cA = \int v\varphi dv$; $c_1 \bar{u}_1 (cA) = \int u_1 \varphi v dv$; $u_1 = u + u_{sl}$ and u, u_{sl}, and u_1 are the liquid, slip, and bubble velocities. The coefficient c_1 depends on the shape of the profile $u_{sl}(v)$:

$$c_1 = \frac{\int u_1 \varphi v dv}{\left(\int \varphi v dv\right) \bar{u}_1}; \quad \bar{u}_1 = \frac{\int u_1 \varphi dv}{\int \varphi dv}. \tag{7}$$

In the transition from (1) to (6) information about the profile $u_{sl}(v)$ is sacrificed, so that c_1 must be given experimentally or the solution (1) must be subjected to simplifying assumptions.

Thus, for $u_1 = \text{const}$ we have $c_1 = 1$. The efflux of vapor mass due to growth of the bubbles during their motion is equal to H_1. We know that new bubbles are formed on irregularities of the walls [10-12]. Recognizing the fact that bubbles arrive at the point (t, x) from all the side walls between x and $x - u_1 t$ and from the bottom for $x - u_1 t \leq 0$, we write the expression for H_1 in the form

$$\begin{aligned} H_1(t, x) = \int\limits_0^{\infty} \int\limits_{x - ut \geqslant 0}^{x} n_c P_c \varphi_c (t', x_0, v_0) \cdot v_x (t, t', x, x_0, v_0)\, dx_0 dv_0 \\ + \int\limits_0^{\infty} n_0 A_0 \varphi_0 v_x dx = A_1 + A_2. \end{aligned} \tag{8}$$

Here $A_2 = 0$ if $x - u_1 t > 0$; $t' = t - \int\limits_{x_0}^{x} dx/u_1$; (x_0, v_0) are the coordinates of the bubble at the instant of breakoff from the wall. Taking the results of [1-9] into account with reference to the variable $\Delta T(t, x)$, we write the bubble growth rate in the form

$$\begin{aligned} v_x (t, t', x, x_0, v_0) = \frac{\alpha(t, x)}{u_1} \left[v^{\frac{1}{2}} \right. \\ \left. + \frac{1}{2u_1} \int\limits_{x_0}^{x} \alpha\left(t - \frac{x - x''}{u_1}, x''\right) dx'' \right]. \end{aligned} \tag{9}$$

Assuming that the bubbles breaking off from the wall are of the same size $\varphi_c = \delta(v - v_0)$, $\varphi_0 = \delta(v - v_0)$, we obtain for H_1

$$\begin{aligned} H_1(t, x) = \left[\alpha(t, x) \int\limits_{x - u_1 t \geqslant 0}^{x} \frac{n_c P_c v_0^{\frac{1}{2}}}{u_1} dx' \right] \\ + \left[\alpha(t, x) \int\limits_{x - u_1 t \geqslant 0}^{x} \frac{n_c P_c}{2u_1^2} \int\limits_{x'}^{x} \alpha\left(t - \frac{x - x''}{u_1}, x''\right) dx'' dx' \right] \\ + \left[\alpha(t, x) \frac{n_0 A_0 v_0^{\frac{1}{2}}}{u_1} \right] + \left[\frac{\alpha(t, x)}{2u_1^2} \int\limits_0^{x} \alpha\left(t - \frac{x - x''}{u_1}, x''\right) dx'' \right] \\ = C_1 + C_2 + C_3 + C_4; \end{aligned} \tag{10}$$

C_1 and C_2 characterize the boiling at the side walls; C_3 and C_4 correspond to the bottom of the volume; and $C_3 = C_4 = 0$ if $x - u_1 t > 0$. We write the continuity and energy equations for the vapor and liquid in the form

$$\begin{gathered} (\rho_1 cA)_t + (\rho_1 u_1 cA)_x = \rho_1 H; \quad (\rho(1 - c)A)_t + (\rho u(1 - c)A)_x = -\rho_1 H; \\ (\rho_1 c h_1 A)_t + (\rho_1 u_1 c h_1 A)_x = q_v + \rho_1 H(h + h_{vap}); \\ (\rho(1 - c)hA)_t + (\rho(1 - c)uhA)_x = q_l - \rho_1 H(h + h_{vap}). \end{gathered} \tag{11}$$

The equations for the mixture can be obtained by adding the phase equations (11). The solutions of the one-dimensional equations (11) are analyzed for certain situations in [13]; here we propose to consider balance methods for calculating the vapor content. To obtain the mass and energy balance for the phases we integrate (11) and the mixture equation with respect to x and allow for the conditions at the discontinuity surfaces $x = x_1$ (free surface of the liquid) and $x = 0$ (bottom of the volume):

$$\dot{x}_1 = \frac{G}{\overline{\rho}} - \frac{j_{\text{vap}}}{\rho} - \frac{u_{\text{s1}} c\rho}{\overline{\rho}}; \quad (GA)_{x=0} = (GA)_0; \tag{12}$$

j_{vap} is the mass flux due to evaporation from the free surface. Now the mass balance for the mixture, liquid, and vapor phases and the energy balance for the mixture can be written in the form

$$\dot{m}_\Sigma \equiv \frac{d}{dt}\int_0^{x_1} \overline{\rho}\, A dx = (GA)_0 - \left(j_{\text{vap}} A \frac{\rho}{\overline{\rho}} \right)_1 - (u_{\text{s1}} c\rho_1 A)_1; \tag{13}$$

$$\dot{m}_l \equiv \frac{d}{dt}\int_0^{x_1} (1-c)\rho A dx = -\int_0^{x_1} \rho_1 H dx - \left(j_{\text{vap}}(1-c) A\right)_1 - ((1-c)\rho u A)_0; \tag{14}$$

$$\begin{aligned} \dot{m}_{\text{v}} \equiv \frac{d}{dt}\int_0^{x_1} c\rho A dx = \int_0^{x_1} \rho_1 H dx - \left(j_{\text{vap}} \frac{c\rho_1 A}{\rho} \right)_1 \\ - (u_{\text{s1}} c\rho_1 A)_1 + (u_{\text{s1}} c\rho_1 A)_0; \end{aligned} \tag{15}$$

$$\begin{aligned} \frac{d}{dt}(c_1 m_{\text{v}} h_1 + c_2 m_l h) \equiv \frac{d}{dt}\int_0^{x_1} (c\rho_1 h_1 + (1-c)\rho h) A dx = \int_0^{x_1} q_\Sigma dx \\ - \left(\left(J_{\text{vap}} A \left(c\rho_1 h_1 + (1-c)\rho h\right)\right) \frac{1}{\rho} \right)_1 + ((c\rho_1 u_1 h_1 + (1-c)\rho u h) A)_0 - (u_{\text{s1}} c\rho_1 h_1 A)_1. \end{aligned} \tag{16}$$

The coefficients c_1 and c_2 depend on the shape of the profile $h(t, x)$, $h_1(t, x)$, and if $h = h(t)$, $h_1 = h_1(t)$, then $c_1 = c_2 = 1$. According to [10-12] $T_1 = T_S$, $h_{10} = c_p T_S + h_{\text{vap}}$. If we postulate as the enthalpy of the vapor $h_1 = h + h_{\text{vap}} = c_p \Delta T + h_{10}$, we find that the relative error $\delta = c_p \Delta T/(c_p T_S + h_{\text{vap}}) \ll 1$ for the majority of substances. For example, $\delta(H_2O) \sim 10^{-4}$, $\delta(N_2) \sim 0.006$, $\delta(O_2) \sim 0.001$, and $\delta(\text{Li}) \sim 10^{-5}$. With this fact in mind, we obtain the following equation from (13)-(16) for the variation of the liquid-phase enthalpy h:

$$m_\Sigma h_t = \int_0^{x_1} q_\Sigma dx - h_{\text{vap}} \int_0^{x_1} \rho_1 H dx - (j_{\text{vap}} A (1-c) h_{\text{vap}})_1. \tag{17}$$

For the variation of the vapor-phase mass, according to (15), we write

$$\begin{aligned} M_1 = \int_0^{x_1} \rho_1 H dx = \rho_1 \Big[\int_0^{u_1 t} \int_0^{x} n_c P_c v_x dx' dx + \int_0^{u_1 t} n_0 A_0 v_x dx \\ + \int_0^{u_1 t} n_c P_c v_0 \Big] + \rho_1 \Big[\int_{u_1 t}^{x_1} \int_{x-u_1 t}^{x} n_c P_c v_x dx' dx + \int_{u_1 t}^{x} n_c P_c v_0 dx \Big]. \end{aligned} \tag{18}$$

For $u_1 t \geq x_1$ the second bracketed expression is equal to zero. In particular, for the case α = const (ΔT = const) we deduce from (18)

$$\begin{aligned} M_1 = \rho_1 \Big[\left(\frac{1}{2} n_c P_c u_1 \alpha v_0^{\frac{1}{2}} \right) t^2 + \left(\frac{1}{12} n_c P_c u_1 \alpha^2 \right) t^3 + \left(n_0 A_0 v^{\frac{1}{2}} \right) t \\ + \left(\frac{1}{4} n_0 A_0 \alpha^2 \right) t^2 + (n_c P_c v_0 u_1) t \Big] + \rho_1 \Big[\left(n_c P_c v_0^{\frac{1}{2}} \alpha u_1 \right) (x_1 - u_1 t) t \\ + \left(\frac{1}{4} n_c P_c u_1 \alpha^2 \right) (x_1 - u_1 t) t + n_c P_c v_0 (x_1 - u_1 t) \Big]. \end{aligned} \tag{19}$$

From the continuity equation (11) for the vapor phase, assuming that $\Delta T = \Delta T(t, x)$ is known, we obtain the expression

$$M_2 = (u_{\text{s1}} c\rho_1 A)_1 - (u_{\text{s1}} c\rho_1 A)_0 = \int_0^{x_1} \int_0^{x} \rho_1 n_c P_c v_x dx' dx$$

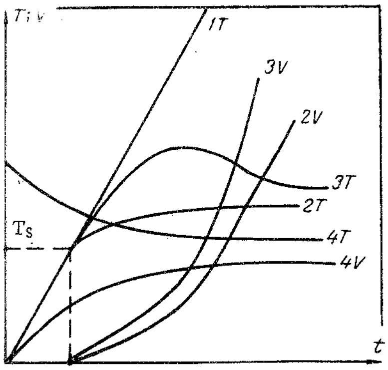

Fig. 1. Distribution of T(t) and $V_V(t)$.

$$+\int_0^{x_1} \rho_1 n_0 A_0 v_x dx + \int_0^{x_1} \rho_1 n_c P_c v_0 dx; \; x_1 \leqslant u_1 t;$$

$$M_2 = \int_{x_1-u_1t}^{x_1} \int_{x-u_1t}^{x} \rho_1 n_c P_c v_x dx' dx + \int_{x_1-u_1t}^{x_1} \rho_1 n_c P_c v_0 dx'; \; x_1 > u_1 t. \tag{20}$$

Comparing (18), (19), and (20), we infer from (15) that a stationary distribution of vapor is established and $\dot{m}_V = 0$ in the volume at the time $t_1 = x_1/u_1$ for ΔT = const, $u = 0$, and $u_1 = u_{s1}$ (the liquid is motionless). But if liquid flows out of the volume $< [(GA)_0 \neq 0]$ or evaporates from the free surface ($j_{vap} \neq 0$), the total vapor mass varies together with the mass of the entire system. We write equations analogous to (13)-(15) for the volume variation of the phases:

$$\dot{V}_V \equiv \frac{d}{dt} \int_0^{x_1} cA dx = \int_0^{x_1} H dx - \left(\frac{j_{vap}}{\rho} cA \right)_1 - (u_{s1} cA)_1 + (u_{s1} cA). \tag{21}$$

For the special case in which the efflux of vapor from the system with the bubbles across the free surface is much smaller than the vapor formation due to bubble growth we obtain [for $j_{vap} = 0$ and $(GA)_0 = 0$]

$$\int_0^{x_1} H dx \gg (u_{s1} cA)_1; \; \dot{V}_V = \int_0^{x_1} H dx; \; \dot{V}_l = -\int_0^{x_1} \frac{\rho_1}{\rho} H dx; \; \dot{m}_\Sigma = 0. \tag{22}$$

The total mass m_Σ of the mixture is left unchanged, but the volume of the two-phase system increases due to the nucleation and growth of bubbles. This situation is possible in the following cases (of practical significance. First a film can form on the free surface, preventing the bubbles from escaping the liquid at the surface, whereupon the bubbles rising from the bottom cluster near the surface and form a froth layer. The boiling of milk is typical. Second, the speed of the bubbles can be rendered small by a large viscosity on the part of the liquid (as in the case of industrial oils): finally, in any liquid where heat sources of great intensity are present and, accordingly, superheating takes place it is possible to encounter vigorous bubble growth, which is not compensated by the escape of vapor across the surface. In every case the total volume of the system increases, and the free surface begins to rise. If the liquid is contained in an open vessel, it can spatter out around the edges. In a closed vessel the pressure begins to increase, possibly resulting in rupture. Here the rate of growth of the vapor-phase volume due to bubble growth is determined from (18) and (19). For ΔT = const and $t > x_1/u_1$ we infer from (19) that the increase of the vapor volume $V_V \sim t$. We write the equation (17) for the enthalpy of the liquid phase under condition (22) for $t > x_1/u_1$:

$$\frac{d\alpha}{dt} = \frac{1}{\beta m_\Sigma} \left(Q_\Sigma - h_{vap} \int_0^{x_1} \rho_1 H dx \right), \tag{23}$$

where $1/\beta = \alpha/c_p\Delta T$ = const; $Q_\Sigma = \int_0^{x_1} q_\Sigma dx$. Substituting H from (10) into (23), we obtain a nonlinear integro-differential equation for $\alpha(t)$:

$$\frac{d\alpha}{dt} = A_0 - A_4 - A_1\alpha - A_2\alpha \int_{t-\frac{x_1}{u_1}}^{t} \alpha(\tau) K_2(t, \tau) d\tau$$

$$-A_3 \int_{t-\frac{x_1}{u_1}}^{t} \alpha(\tau) K_3(t, \tau) d\tau;$$

$$A_0 = \frac{Q_\Sigma}{\beta m_\Sigma}; \; A_4 = \frac{h_{vap} \rho_1 n_c P_c v_0 x_1}{\beta m_\Sigma}; \; A_1$$

$$= \frac{h_{vap}}{\beta m_\Sigma} \left(\frac{\rho_1 n_c P_c v_0^{\frac{1}{2}} x_1^2}{2u_1} + \frac{\rho_1 n_0 A_0 v_0^{\frac{1}{2}} x_1}{u_1} \right);$$

$$A_2 = \frac{h_{vap}\rho_1 n_c P_c}{2\beta m_\Sigma u_1};\; A_3 = \frac{h_{vap}\rho_1 n_0 A_0}{2u_1\beta m_\Sigma};\; K_2 = \\ = \frac{1}{2}(x_1 + u_1(t-\tau))^2;\; K_3 = x_1 - u_1(t-\tau). \tag{24}$$

The coefficients A_i are functions of ΔT. According to [2, 4, 6, 10, 11] we can assume that $A_i = \bar{A}_i\alpha^{n_i}$. For $t \gg x_1/u_1$ the quantity α can be taken outside the integral in (24):

$$\frac{d\alpha}{dt} = B_0 + B_4\alpha^{n_4} + B_1\alpha^{n_1+1} \\ + B_2\alpha^{n_2+2}. \tag{25}$$

In particular, for $n_i = 0$ we obtain

$$\alpha(t) = D_1 \\ + \frac{\exp(D_3 t)}{(\alpha_0 - D_1)^{-1} + D_2(1 - \exp(D_3 t))}; \tag{26}$$

$$D_1 = \frac{-B_1 + \sqrt{B_1^2 - 4B_5 B_2}}{2B_2};\; D_2 = \frac{B_2}{B_1 + 2D_1 B_2};$$

$$D_3 = B_1 + 2D_1 B_2;\; B_5 = B_0 + B_4.$$

It is evident from (23)-(26) that the superheat $\alpha(t)$, varying as a function of B_0, B_i, and α_0, at first increases and then decreases to D_1. The solution of (23) is given in Fig. 1, in which curve 1 corresponds to heating of a single-phase liquid, curves 2 and 3 to the boiling of a two-phase liquid, and curve 4 to boiling with an abrupt pressure drop in the volume, this effect being equivalent to abrupt superheating of the liquid. Curve 2 typifies boiling with a "low" heat source, for which the superheat increases monotonically, while curve 3 typifies boiling with a "strong" heat source, for which the superheat at first increases and then subsides.

In the bubbling of gas through holes in the walls v_0, n_c, and n_0 are independent of the superheat ΔT, but the growth of the bubbles during their motion is a function of ΔT: $[v = v(\Delta T)]$. Under the condition $p_v \gg p_g$ (p_v and p_g are the partial pressures of the vapor and gas in the bubbles) the bubble growth can be determined from (3). It is clear that $p_v \gg p_g$ for $t \gg 1$, for $x \gg 1$, or for $\Delta T \gg 1$. Under these conditions the general equations (8)-(26) describe bubbling processes in a liquid.

NOTATION

δ	is the delta function;
θ	is the unit Heaviside function;
1, Σ	are subscripts referring to the vapor and mixed phases;
$G = \rho_1 u_1 c + \rho u(1-c)$	is the mass flow rate of the mixture;
$\bar{\rho} = \rho_1 c + \rho(1-c)$	is the average density of the mixture
m_Σ, m_v, m_l, h_Σ, h_1, h	are the masses and enthalpies of the mixture, vapor, and liquid, respectively;
h_{vap}	is the heat of vaporization;
$\Delta T = T - T_s$	is the superheat relative to saturation;
Ja	is the Jacob number;
S	is the total surface area of the walls;
V_v	is the vapor volume;
T_s	is the saturation temperature;
h_1, h_{10}	refer to the enthalpy of the vapor.

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